Conditions of the uniform convergence of empirical averages to their expectations

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Abstract:

Some major theoretical and empirical results of the domain in the past

The large number law asserts that empirical averages of a random function converge to its expectation. But, is it true if we calculate empirical averages over a set of random functions using the same sample sequence (in the sense of uniform convergence)? It appears that in general it is not true. Sufficient conditions for the uniform convergence based on VC dimension were found. Also necessary and sufficient conditions based on entropy properties of a set of functions were proven. In the report I shall talk about properties of random function sets for which the uniform convergence holds or not holds.

Some unresolved problems for future.

It would be very interesting to convert the asymptotic results to some estimates valid for finite sample sequences, and apply them to the problem of optimal complexity choice in machine learning algorithms.

Keywords:

Large number law, uniform convergence, VC dimension, entropy properties